4.2 Notes and Examples

Name: Block: Seat:

1. Visualize estimating the area under a curve here: https://www.desmos.com/calculator/bigznbc4sf
2. After ΔMath "4.2 Riemann Examples" Answer the following
   (a) A left Riemann sum is an underestimate if the curve is:

   (b) A left Riemann sum is an overestimate if the curve is:

   (c) A right Riemann sum is an underestimate if the curve is:

   (d) A right Riemann sum is an overestimate if the curve is:

   (e) A trapezoidal Riemann sum is an underestimate if the curve is:

   (f) A trapezoidal Riemann sum is an overestimate if the curve is:

3. A car is traveling so that its speed is never decreasing during a 12 second interval. The speed at various moments in time is listed below:

Time in seconds	0	3	6	9	12
Speed in ft/sec	30	37	45	54	65

(a) Sketch a possible graph of this function

(b) Estimate the distance traveled by the car during the 12 seconds by finding the areas of four rectangles at the heights of the **left** endpoints. This is called a **Left Riemann sum**

$\boxed{A_{rect} = b \cdot h}$

(c) Estimate the distance traveled by the car during the 12 seconds by finding the areas of four rectangles at the heights of the **right** endpoints. This is called a **Right Riemann sum**

Time in seconds	0	3	6	9	12
Speed in ft/sec	30	37	45	54	65

(d) Estimate the distance traveled by the car during the 12 seconds by finding the areas of two rectangles at the heights of the **midpoints**. This is called a **Midpoint Riemann sum**

(e) Estimate the distance traveled by the car during the 12 seconds by finding the areas of four trapezoids. This is called a **Trapezoidal Riemann sum**

$$\boxed{A_{trap} = \frac{1}{2}(f(a) + f(b)) \cdot \Delta x}$$

To get better and better estimates, we need to bring in limits of sums, so next we review sums in sigma notation.

Final value for k.

Sum of all terms $\sum_{k=m}^{n} f(k)$ describes each term

starting value for k

4. Write in Sigma notation:
   (a) $1^2 + 2^2 + 3^2 + 4^2 + 5^2$

   (b) $2 - 4 + 6 - 8 + 10$

   (c) Calculator tricks:
   $\boxed{\text{LIST}}$ $\boxed{\text{OPS}}$ 5:seq $\boxed{\rightarrow}$ L_1
   $\boxed{\text{LIST}}$ $\boxed{\text{MATH}}$ 5:sum L_1

THEOREM 4.2 Summation Formulas

1. $\sum_{i=1}^{n} c = cn$, c is a constant

2. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

3. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

4. $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

5. Try these
   (a) $\sum_{i=1}^{10} 2i + 1$

   (b) $\sum_{i=1}^{9} 3i$

6. Show $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i}{n}\right)\left(\frac{2}{n}\right) = 2$

   (a) multiply:

   (b) factor out $\frac{4}{n^2}$

   (c) substitute the sum formula:

   (d) take the limit

   (This turns out to be the area under the curve $f(x) = x$ from $x = 0$ to $x = 2$ which we will later write as $\int_0^2 x\,dx$... more about this in section 4.3)

7. Show $\lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + \frac{2i}{n}\right)^2 \left(\frac{2}{n}\right) = \frac{26}{3}$

   (a) expand the binomial and distribute $\frac{2}{n}$

   (b) Make it about three sigmas, factor out the constants ($\frac{2}{n}$, $\frac{8}{n^2}$, $\frac{8}{n^3}$)

   (c) Substitute the sigmas for the formulas, cancel out some n's

   (d) Take the limit as $n \to \infty$, add the three fractions

(This turns out to be the area under the curve $f(x) = x^2$ from $x = 1$ to $x = 3$ which we will later write as $\int_1^3 x^2\,dx$... more about this in section 4.3)

Sums of Rectangles and Trapezoids to estimate the area under the curve

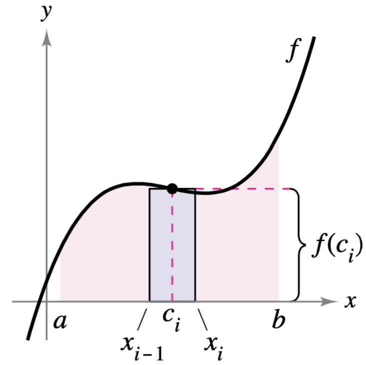
Definition of the Area of a Region in the Plane

Let f be continuous and nonnegative on the interval $[a, b]$. (See Figure 4.13.) The area of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is

$$\text{Area} = \lim_{n \quad \infty} \sum_{i=1}^{n} f(c_i)\,\Delta x$$

where $x_{i-1} \le c_i \le x_i$ and

$$\Delta x = \frac{b-a}{n}.$$



The width of the ith subinterval is $\Delta x = x_i - x_{i-1}$.
Figure 4.13

$\boxed{A_{rect} = b \cdot h}$

8. $\displaystyle\sum_{i=0}^{n} f(c_i) \cdot \Delta x$, where:

(a) Width $\Delta x = \dfrac{b-a}{n}$

(b) Left endpoint rule for $f(c_i) : f(a + (i-1)\Delta x)$

(c) Right endpoint rule for $f(c_i) : f(a + (i)\Delta x)$

(d) Midpoint endpoint rule for $f(c_i) : f(a + (i - \frac{1}{2})\Delta x)$